Practical Measures,
Routines & Representations

Thanks for expressing interest in our task analysis tool!

In this document, you'll find an annotated version of the tool, which includes the research informing the tool and information for use of the tool.

Please note that we are in the process of refining these tools. It is important to us that we learn from those who are using them. We are currently operating under a Creative Commons license. As such, we ask that you track and share any revisions you make to the tool. If you'd like to read more about how the tool was developed, to find the most recent version of this tool, or to download other tools for instructional improvement, visit http://pmr2.org.

A word of caution: This tool is intended to support inquiry about teaching and to inform instructional improvement efforts. It is not appropriate to use this tool to evaluate teachers' instruction.

Thank you!

The PMR2 Team

RIGOR OF THE TASK

Annotated Rigor of the Task Analysis Tool

Intended Use:
This tool is intended to help educators assess the level of rigor or cognitive demand of the mathematical tasks as written and before they are implemented in the classroom. The leading question to guide analysis is: What do students have to do to successfully complete the task?

Math education research indicates that the rigor of the tasks that teachers select as the basis for their instruction influences all phases of lessons and thus students' opportunities to learn (e.g., Boston, 2012; Stein & Lane, 1996; Stein, Grover, & Henningen, 1996). Building on the work of Mary Kay Stein, Melissa Boston, and colleagues, the tool distinguishes between three level of rigor: Using Procedures, Making Sense of Procedures, and Problem Solving.

- Using Procedures: Students can solve the task by using a previously taught procedure and do not need to explain or demonstrate why the procedure works in order to be successful.

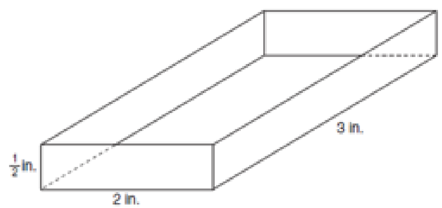
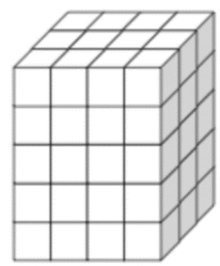
- Making Sense of Procedures: Students have to explain and/or demonstrate why a procedure works in order to be successful. The cognitive demand of these tasks is higher than Using Procedures because students have to demonstrate that they understand the procedure, often by using models.

- Problem Solving: Students have to figure out which procedures to use by analyzing the task and identifying underlying mathematical relations. The cognitive demand of these tasks is higher than Making Sense of Procedures because students have to analyze tasks mathematically in order to figure out how they can be solved.

As tasks increase in rigor, they admit a wider range of solution strategies, which in turn provide a basis for productive classroom discussions that focus on significant mathematical ideas.

Analyzing the Rigor of the Task

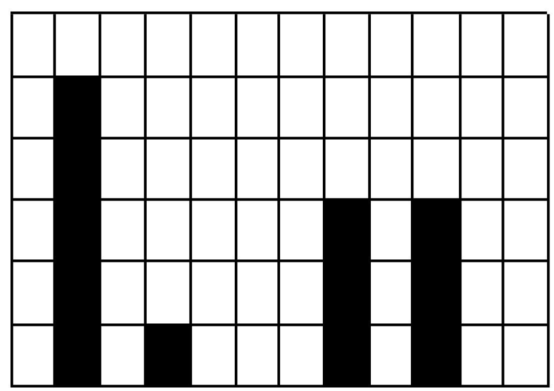
Leading question: What do students have to do to successfully complete the task?

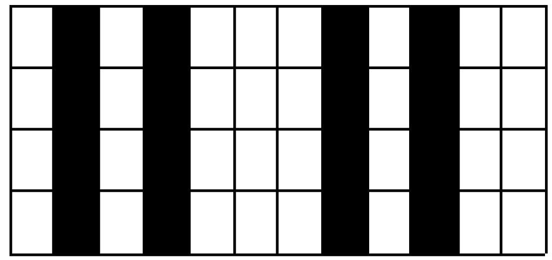
	Using Procedures	Making Sense of Procedures	Problem Solving
Description	The student has to use a procedure to solve a familiar type of task.	The student has to use a procedure to solve a task and demonstrate or figure out why the procedure works.	The student has to analyze the problem in order to figure out what procedure to use.
Measure-ment example	Mr. Munoz drew a box in the shape of a rectangular prism, as shown below. What is the volume of the box? 	Which expression(s) below can you use to determine the volume? Explain why that works. 20 x 12 15 x 3 12 x 5 	An 80-gallon bathtub is being filled at a rate of 15 gallons every 2 minutes. At this rate, how many minutes will it take to fill this bathtub ¾ full?
Rationale	The task requires students to perform a procedure (find the volume of a rectangular prism). Does not require the procedure to be connected to meaning or understanding.	The task requires students to think about volume as more than the expression length times width times height. Because the expressions given have only two numbers, students are given the opportunity to relate the three-dimensional prism with thinking about volume as layers. The explanation for	There is not a predictable procedure to use in this situation. Students will need to reason about what the different values mean and how to solve it. While there is one correct answer, there are multiple ways students can get to that answer.

		why the expression works helps students making sense of the volume equation.	

<table>
<tr><th></th><th>Using Procedures</th><th>Making Sense of Procedures</th><th>Problem Solving</th></tr>
<tr><td>Description</td><td>The student has to use a procedure to solve a familiar type of task.</td><td>The student has to use a procedure to solve a task and demonstrate or figure out why the procedure works.</td><td>The student has to analyze the problem in order to figure out what procedure to use.</td></tr>
<tr><td>Data Example</td><td>Example 1: Calculate the mean of each set of numbers.

a. 6, 2, 1
b. 3, 5, 5, 11
c. 4, 2, 6, 13, 10

Example 2: The table below shows the number of passengers that travel through the five busiest airports in the world each year. What is the mean number of passengers in these airports?
<table><tr><th>Airport</th><th>Passengers (in millions)</th></tr><tr><td>Atlanta, Hartsfield</td><td>80.2</td></tr><tr><td>Chicago, O'Hare</td><td>72.1</td></tr></table></td><td>The pairs of numbers below represent the heights of 2 columns of cubes that need to be made equal in height. On grid paper, sketch the columns before and after they made equal (see example below for the number pair 9 and 5). Write a statement under the sketch that explains how your method of equalizing is related to finding the mean of the two numbers.
</td><td>The best player on a basketball team made 12 out of 20 free throws in the first game and 14 out of 25 free throws in the second game. If she takes 20 free throws in the third game, how many does she need to make to average 60% free throw shooting across the three games? Explain your reasoning.</td></tr>
</table>

<table>
<tr>
<td></td>
<td>
<table>
<tr><td>Los Angeles</td><td>68.5</td></tr>
<tr><td>London, Heathrow</td><td>64.6</td></tr>
<tr><td>Dallas/Ft. Worth</td><td>60.7</td></tr>
</table>
</td>
<td>

9 5 7 7
Before After
1. 14 and 8 2. 16 and 7
3. 7 and 12 4. 13 and 15
</td>
<td rowspan="2">This task requires students to go beyond simply using the procedure to calculate the average and to make deeper sense of the nature of averages. Students must analyze several sets of numbers to determine exactly how to calculate an overall average of 60% free throw shooting. There are multiple ways for students to solve this task. Students are also asked to explain their reasoning.</td>
</tr>
<tr>
<td>Rationale</td>
<td>The task requires students to perform a procedure (calculate the mean of a set of numbers). Does not require the procedure to be connected to meaning or understanding. Even though Example 2 is situated in a real-world situation, the student can solve the problem using a procedure.</td>
<td>The task requires students to draw models and explain how “leveling-off” columns of numbers is related to calculating the mean. Students are asked to perform a procedure (calculate the mean of two numbers), but the models serve to demonstrate how and why the procedure for calculating the mean works.</td>
</tr>
</table>

	Using Procedures	Making Sense of Procedures	Problem Solving
Description	The student has to use a procedure to solve a familiar type of task.	The student has to use a procedure to solve a task and demonstrate or figure out why the procedure works.	The student has to analyze the problem in order to figure out what procedure to use.

Algebra example	For each pair of points listed below, (a) Plot the points on a coordinate grid and draw a line through them. (b) Find the slope of the line. (c) Find the y-intercept from the graph. (d) Use your answers from parts (b) and (c) to write an equation for the line. (e) Find one more point that lies on that line. 1. (0,0) and (3,3) 2. (-1,1) and (3,-3) 3. (0,-5) and (-2,-3) 4. (3,6) and (5,6)	A class is having a walk-a-thon to raise money, and students find sponsors to donate. ● Leanne's sponsors will pay \$10 regardless of how far she walks. ● Gilberto's sponsors will pay \$2 per km. ● Alana's sponsors will make a \$5 donation plus 50¢ per km. (a) Make a table for each student's plan showing the amount of money each of his or her sponsors will owe if s/he walked from 0 to 6 km. (b) Graph the 3 pledge plans on the same coordinate axes. (c) Write an equation for each pledge plan. Explain what information each number and variable in your equation represents. (d) What patterns do you observe in the table for each pledge plan? How do these patterns appear in the graph? In the equations?	A class is having a walk-a-thon to raise money, and students find sponsors to donate. ● Leanne's sponsors will pay \$10 regardless of how far she walks. ● Gilberto's sponsors will pay \$2 per km. ● Alana's sponsors will make a \$5 donation plus 50¢ per km. Which student will make the most money? Explain your reasoning.
Rationale	The tasks can be solved by substituting numbers into a formula. Students are required to perform a procedure and show their steps, but the procedure is not connected to meaning or understanding. The cognitive demand of this task is low because students are told in each task exactly what to do.	This task provides a real-world example of linear relationships, with students directed to make a table, graph, and equation for each relationship. Students are asked to relate these algebraic representations with one another. Part (d) in this task is key for students to help students make sense of procedures; without part (d), this would be considered "Using Procedures."	This task is considered higher cognitive demand than the "Making Sense of Procedures" example because students have to analyze the problem to figure out how to decide which student would make. Students are not directed to use specific strategies but do have to explain their reasoning.

References:

Boston, M. D. (2012). Assessing the quality of mathematics instruction. Elementary School Journal, 113(1), 76-104.

Stein, M. K., Grover, B., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. American Educational Research Journal, 33, 455–488.

Stein, M. K., & Lane, S. (1996). Instructional tasks and the development of student capacity to think and reason: An analysis of the relationship between teaching and learning in a reform mathematics project. Educational Research and Evaluation, 2(1), 50-80.